No - PGD/Maths/25/604
Dated: 01-08.25

DEPARTMENT OF MATHEMATICS

SYLLABUS : ENTRENCE TEST FOR Ph. D. ADMISSION IN MATHEMATICS

1) REAL ANALYSIS

Elementary set theory, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum and infimum, Resume of sequences, series and Riemann integration, continuity, uniform continuity, differentiability, Fundamental theorem of integral calculus, classes of R-integrable functions, Functions of bounded variation, Cauchy's general principle of uniform convergence, uniform convergence and integration, uniform convergence and differentiation, weierstras theorem.

2) COMPLEX ANALYSIS

Complex numbers and functions. CR equations, analytic functions, entire functions, the order of an entire function. Necessary and sufficient condition for analyticity, harmonic functions, harmonic conjugate, contour integration. Cauchy integral formula/ theorem, Liouvilles theorem, Taylor's and Laurent's theorems, classification of singularities, Riemann's theorem, weierstras theorem on essential singularity. Calculus of residues, Cauchy's residue theorem, Integration by the method of residues, maximum/ minimum modulus theorem. Schwarz lemma. Power series, Hadamard formula for the radius of convergence, a power series represents an analytic function within the circle of convergence. Rouch's theorem, the fundamental theorem of algebra. Morea's theorem, Hurwitz theorem.

3 Linear Algebra

Vector spaces over real and complex fields, linear dependence and independence, subspaces, bases, dimensions. Linear transformations, rank and nullity, matrix representation of linear transformations, linear functional. Algebra of matrices: row and column reduction, echelon form, congruence and similarity, rank, inverse, solutions of linear systems. Eigenvalues and eigenvectors, characteristic polynomial, Cayley-Hamilton theorem. Special matrices: symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal, unitary, and their eigenvalues, quadratic forms.

4 Number theory:

Divisibility and Prime Numbers: Division algorithm, Euclidean algorithm, GCD, LCM, prime numbers, fundamental theorem of arithmetic, congruence's, Chinese remainder theorem., Modular Arithmetic: properties of congruence's, linear congruence's, Fermat's little theorem, Euler's theorem, Wilson's theorem., Euler's totient function, Mobius function, Mobius inversion formula, sigma function, number of divisors function, Diophantine Equations, Quadratic Residues and Reciprocity (Legendre symbol, quadratic reciprocity law, Jacobi symbol), and Continued Fractions.

5 **Abstract Algebra**

GROUPS: Groups, subgroups, abelian groups, normal subgroups, quotient groups, homomorphism of groups, cyclic groups, Structure theorem for cyclic groups, permutation groups, Sn , An., class equations, automorphisms, inner automorphisms. Cauchy's and sylow's theorem for Abelian groups. Cayley's theorem. Sylow's theorem and Cauchy's theorem. Finite Abelian groups, Fundamental theorem on finite Abelian groups. Composition series. The Jordan-Holder theorem for finite groups.

RINGS: Definition and examples of rings. Integral domains, ideals, Principal ideal, Prime ideals and maximal ideals, Nil ideal, Radical ideal, Annihilator ideal, quotient rings, UFD, PID, ED, Fields of quotients of an integral domains, Polynomial rings, Irreducible polynomials, Einstein's criterion.

FIELDS: Fields, finite fields and sub fields, Prime fields and their structure. Extensions of fields. Algebraic numbers and algebraic extensions of a field. Galois Theory.

6 **MEASURE THEORY**

σ- Algebra , Measure on σ- algebra, Measures of sequences of sets, Measureable spaces and measure space. Outer measures, regular outer measure, metric outer measure, construction of outer Measure.

Lebesgue outer measure on R, properties of Lebesgue measure spaces, translation invariances of Lebesgue measure. Existence of non-Lebesgue measureable sets. Regularity of Lebesgue outer measure. Cantor ternary set and cantor function. Relation between Lebesgue and Borel measurability, completion of measure space. Completion of Borel measure space to the Lebesgue measure space.

Measureable functions , Operation with measureable function, equality a.e. , Sequence of measureable functions, continuity and Borel Lebesgue measureability of functions on R, integration of simple functions, Lebesgue integral of non-negative and measureable functions. Properties of Lebesgue integrals. Convergence a.e. , Almost uniform convergence, Egroff's theorem, convergence in measure, convergence in mean. Cauchy sequence in measure, relation among various convergence types. Fatou's lemma, Lebesgue monotone convergence theorem. Lebesgue dominated theorem.

DIFFERENTIAL EQUATIONS

First order ODE's, integrating factors, Wronskian of solutions, initial value problem, singular solutions, P-discriminant, C-discriminant. Equations of the second degree with constant coefficients. Total differential equation. Pdx+Qdy+Rdz = 0. Necessary and sufficient conditions that such a differential equation may be integrable. Partial differential equation of the first order, Lagrange's and charpits method for solving first order PDE's, classification of second order PDE's.

7 **TOPOLOGY**

Metric spaces: Definition and examples, open sets, completeness, convergence, continuous mapping, completion of a metric space, Cantor's intersection theorem. Contraction mapping, Banach's contraction Principle. Topological spaces: Definition and examples, Elementary properties, Kuratowski's axioms, continuous mappings and their characterisation. Bases and subbases, concept of first countability, second countability, separability, Tychnoffs theorem, Lebasgue's covering lemma, continuous maps on compact spaces, Separation axioms, compactness and connectedness in topological spaces.

8 FUNCTIONAL ANALYSIS:

BANACH SPACES, Definition and examples, Quotient spaces, Dual of a normed linear space. Dual spaces, Hahn Banach Theorem, closed graph theorem, open mapping theorem. HILBERT SPACES, Definition and examples, Cauchy-Schwarz inequality Bessel's inequality, orthonormal systems. Riesz representation theorem, inner product spaces, Adjoint of a Hilbert space, Normal operators.

9 DIFFERENTIAL GEOMETRY:

Curves with torsion, arc length, curvature, S-Frenet formulae, spherical curvature, spherical indicatrices, Involutes and evolutes, helix, Bertrand curves. Helix, Envelopes of one and two parameter family of surfaces, Developable surfaces, Developable associated with a curve. Curvilinear coordinates. Fundamental magnitudes of first and second order, the two fundamental forms; curvature of normal section, Meunier's theorem.

(HOD Mathematics)

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